

SOLUTION SET VIII

EXERCISE VIII.1 : ACTIVE AND REACTIVE POWER

A. $\operatorname{tg}\varphi = Q_s/P_s \Rightarrow \varphi = \underline{33,69^\circ}$

B. $S = \sqrt{P_s^2 + Q_s^2} = U_s \cdot I \Rightarrow I^2 = \frac{P_s^2 + Q_s^2}{U^2} = \underline{83,6 \cdot 10^3 \text{ A}^2}$

Moreover, the powers are written :

$$P_{\text{ch}} = P_s - R \cdot I^2 = \underline{2,8 \text{ MW}} \quad \text{and :} \quad Q_{\text{ch}} = Q_s - X \cdot I^2 = \underline{0,746 \text{ M var}}$$

C. The reactive power is positive, both at the source and the load. At the source, the voltage is ahead of the current of $33,69^\circ$ (point A.). At the level of the load, the voltage U_{ch} is ahead of the current of only $14,92^\circ$:

$$\operatorname{tg}\varphi_{\text{ch}} = Q_{\text{ch}}/P_{\text{ch}} \Rightarrow \varphi_{\text{ch}} = \underline{14,92^\circ}$$

So the phase difference between the two voltages is : $\delta\varphi = \varphi - \varphi_{\text{ch}} = \underline{18,77^\circ}$

EXERCISE VIII.2 : ACTIVE AND REACTIVE POWER

A. $R = |Z| \cdot \cos \varphi = \underline{28,58 \Omega} \quad X = |Z| \cdot \sin \varphi = \underline{16,5 \Omega}$

B. $I = U/|Z|$

$$\Rightarrow P = U \cdot I \cos \varphi = \frac{U^2}{|Z|} \cdot \cos \varphi = \underline{1,388 \text{ kW}} \quad Q = U \cdot I \sin \varphi = \frac{U^2}{|Z|} \cdot \sin \varphi = \underline{0,801 \text{ k var}}$$

$$S = U \cdot I = \frac{U^2}{|Z|} = \underline{1,603 \text{ kVA}}$$

EXERCISE VIII.3 : APPARENT POWER

A. For system 1, we have : $S_1 = U \cdot I_1 = \underline{1,84 \text{ kVA}}$

$$\cos \varphi_1 = P_1/S_1 \Rightarrow \varphi_1 = \pm 45,05^\circ \quad \Rightarrow \quad Q_1 = U \cdot I_1 \sin \varphi_1 = \underline{\pm 1,302 \text{ k var}}$$

For system 2, we have: $S_2 = U \cdot I_2 = \underline{2,30 \text{ kVA}}$

$$\cos \varphi_2 = P_2/S_2 \Rightarrow \varphi_2 = \pm 42,34^\circ \quad \Rightarrow \quad Q_2 = U \cdot I_2 \sin \varphi_2 = \underline{\pm 1,549 \text{ k var}}$$

Remember that the $\cos \varphi$ does not tell us anything about the nature of the reactance (capacitive or inductive).

B. For both systems in parallel, it is interesting to represent each system as a resistance R_{pk} in parallel with a reactance X_{pk} ($k=1 ; 2$). We have :

$$R_{pk} = U^2/P_k \quad \text{et} \quad X_{pk} = U^2/Q_k \Rightarrow P_p = \frac{U^2}{R_{p1}} + \frac{U^2}{R_{p2}} \quad \text{and} \quad Q_p = \frac{U^2}{X_{p1}} + \frac{U^2}{X_{p2}}$$

P_p and Q_p being the active and reactive powers for all of both systems in parallel.

$$\Rightarrow S_p = \sqrt{P_p^2 + Q_p^2}$$

Numerical application :

For impedances of the same nature (reactive powers of the same sign), we find :

$$\begin{aligned} R_{p1} &= 40,69 \, \Omega & X_{p1} &= 40,63 \, \Omega \\ R_{p2} &= 31,12 \, \Omega & X_{p2} &= 34,15 \, \Omega \\ P_p &= 3 \text{ kW} & Q_p &= 2,85 \text{ kvar} \\ \Rightarrow S_p &= \underline{4,14 \text{ kVA}} \end{aligned}$$

For impedances of different natures (negative reactive power for system 2), we find :

$$\begin{aligned} R_{p1} &= 40,69 \, \Omega & X_{p1} &= 40,63 \, \Omega \\ R_{p2} &= 31,12 \, \Omega & X_{p2} &= -34,15 \, \Omega \\ P_p &= 3 \text{ kW} & Q_p &= -0,247 \text{ kvar} \\ \Rightarrow S_p &= \underline{3,01 \text{ kVA}} \end{aligned}$$

C. For both systems in series, it is interesting to represent each system in the form of a resistance R_{sk} in series with a reactance X_{sk} ($k=1 ; 2$). We have :

$$R_{sk} = P_k/I_k^2 \quad \text{et} \quad X_{sk} = Q_k/I_k^2$$

The total impedance formed by both systems connected in series :

$$\begin{aligned} \underline{Z}_s &= (R_{s1} + R_{s2}) + j(X_{s1} + X_{s2}) \quad \text{et} \quad Z_s \equiv |\underline{Z}_s| = \sqrt{(R_{s1} + R_{s2})^2 + (X_{s1} + X_{s2})^2} \\ \Rightarrow I_s &= U/Z_s \\ \Rightarrow S_p &= U \cdot I_s = U^2/Z_s \end{aligned}$$

Numerical application :

For impedances of the same nature (reactive powers of the same sign), we find :

$$\begin{aligned} R_{s1} &= 20,31 \, \Omega & X_{s1} &= 20,35 \, \Omega \\ R_{s2} &= 17 \, \Omega & X_{s2} &= 15,49 \, \Omega \\ Z_s &= 51,74 \, \Omega & \Rightarrow S_s &= \underline{1,02 \text{ kVA}} \end{aligned}$$

For impedances of different natures (negative reactive power for system 2), we find :

$$\begin{aligned} R_{s1} &= 20,31 \, \Omega & X_{s1} &= 20,35 \, \Omega \\ R_{s2} &= 17 \, \Omega & X_{s2} &= -15,49 \, \Omega \\ Z_s &= 37,63 \, \Omega & \Rightarrow S_s &= \underline{1,41 \text{ kVA}} \end{aligned}$$